$AN = AM\cos\beta = r\cos\beta - a\sin\beta$.

Volume of $ADEG = 2\pi ar^2$.

Volume of $EGF = 2\int_0^{2r}\int_0^{\sqrt{(2rx-x^2)}}\int_0^{x\tan\beta} dxdydz = \pi r^3\tan\beta$.

[This result comes easily without the calculus].

Substituting in equation (1), transposing, dividing through by $\cos\beta$, rearranging, and clearing of fractions,

$$5r^2\tan^3\beta + 16ar\tan^2\beta + 2(8a^2 - 3r^2)\tan\beta - 16ar = 0.$$

Substituting in this, $\frac{1}{2}h - r\tan\beta$ for a, and reducing,

$$5r^2\tan^3\beta - 8rh\tan^2\beta + 2(5r^2 + 2h^2)\tan\beta - 8rh = 0,$$

from which β may be found.

AVERAGE AND PROBABILITY.

62. Proposed by O. S. KIBLER, Superintendent of Schools, Middleburg, O.

A bag contains any number of balls, which are equally likely to be white or black; one is drawn and found to be white. Show that the chance of drawing another white one, the first ball not being replaced, is two-thirds. [From *C. Smith's Treatise on Algebra*, page 615].

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

If m balls are drawn and turn out white, the chance that n others drawn will be white is: $p = [(m+1)/(m+n+1)]$. In the problem, $m = n = 1$.

$\therefore p = \frac{2}{3}$. Otherwise, $p = \int_0^1 x^2dx / \int_0^1 xdx = \frac{2}{3}$.

This is the simplest case of the article on page 107, No. 4, Vol. II. of the MONTHLY.

64. Proposed by REV. W. A. WHITWORTH, A. M.

O is a given point within a triangle; P is a random point within the same. The line through O and P is produced so as to divide the triangle into a trapezium and a triangle. Find the average area of this triangle. [From the *Educational Times*, London, Eng.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

Let $\triangle$ be the average area required, $\triangle_1$ the average area of BFG; (m, n) coördinates of O. Then the coördinates of E are $(b, bn/m)$; of D, $[b^2n/(ab-am+bn), abn/(ab-am+bn)]$.

Area $ABC = \frac{1}{2}ab\sin C$.

Area $ACO = \frac{1}{2}bn\sin C$.

Area $ADO = bn(am-bn)\sin C/[2(ab-am+bn)]$.

Area $COE = bn(b-m)\sin C/(2m)$.

Area $DOEB = ABC - (ACO + ADO + COE)$.